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# A fuzzy based integrated model for identification of vital node in terrorist network using logarithmic concept

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**Abstract.** Analysis of the terrorist network is a process to analyze or deriving Useful information from the available network data. Ranking The Terrorist nodes within a terrorist network in identifying the most influential node is essential for the elaboration of Covert network mining. The purpose of this paper is to implement an approach of two dimensional criteria weight determination along with logarithmic concept implementation for vital node investigation in term of their influential ability. Betweenness, Closeness, Eigenvector, Hub, In-degree, Inverse closeness, Out-degree and Total degree considered as criteria and terrorist involved in 9/11 terrorist attack considered as alternatives used to formulate a decision problem. Although an integrated approach of Fuzzy based subjective-Aggregation concept based objective criteria weight determination and Ranking alternatives by the implementation of the logarithmic concept is used to solve multi criteria decision problems in order to show the application of most centralized node identification process which can be obtained easily by classification and selection problem solution using multiple criteria and alternatives.

**Keywords:** Social network, Social Network Analysis(SNA), Terrorist network analysis(TNS), Fuzzy based criteria weight, Aggregation criteria weighting, Approach of logarithmic concept(APLOCO), Multi Criteria Decision Making(MCDN), Centrality measures, Most influential node

## 1. Introduction

In today's time, if asked about the biggest problem of the country and abroad, then every person will say that terrorism has happened. Terrorism has kept our country and society in such a way that even after lakhs of efforts, it is not getting separated from the root. The more we press it, the more it appears in macabre form in front of us. Terrorism is an illegal act which aims to create fear of violence within common people. If a person or a group of human beings meet together and there is a moment of violence, Riots, Rape, Kidnapping or Bomb blast then it is called terrorism. On 26

November 2008, 166 people along with 26 foreign nationals were killed in Mumbai's Terrorist Attack. 10 attackers entered Mumbai from Pakistan by boat via sea route and three were operating from Pakistan.

A terrorist organization has a defining structure and any main objective, with the help of which any terrorist organization can be easily identified. In the last few years, terrorist organizations have become opaque and more spreading. The hidden structure in any terrorist network is to get information called Social Network Analysis (SNA). This article will explain about the Social Network Analysis as well as it will also explain how to use SNA to investigate terrorist organization activities and other covert networks information.

In this study, most centralized nodes have been identified in terrorist networks. The most centralized node is a node that can easily spread up any

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information in any network efficiently. It has been seen, employing a single criteria or measure to identify key or central node is not sufficient to provide a more accurate result because all criteria have their own perspective to generate the results, for example, if we use degree centrality measure to rank the node then it will focus on how many numbers of links are getting incident at any node and how many numbers of links are going out from that node, on the basis of that degree centrality will tell us the rank of nodes in any network similarly between Centrality will check the number of occurrence of a node in between of shortest path of all pairs of nodes. That is why it has been identified most influential nodes in a network by using multiple centrality criteria like betweenness centrality, closeness centrality, eigenvector centrality, performance Centrality and efficiency Centrality, which will give us more accuracy while finding the result.

Multi-Criteria Decision Making (MCDM) is a rapidly popular method with the help of which Decision Makers can develop a composite process, which Reduces Uncertainty and Supports Decision. This method is popular because it considers an unbiased consensus and it also brings out solutions to those difficult problems in which any problem appears in the differential angle; Alternatives are in different situations, high conflict and more vagueness.

MCDM problems can be solved in two basic steps. First is a selection of weight of criteria. Literature shows that there are many approaches or methods to calculate weight like AHP [2], Fuzzy AHP [5], Entropy [26], CRITIC method [27], SAW [6], Delphi technique [28], FUCOM [29] etc. Many of them required to satisfy the belief and certain state while calculating the criteria weight. Few of them asked for investigating the importance of order before weight calculation and others take the expert suggestion and required the hierarchical structure of criteria order.

In this paper, we focus on the two most important factors necessary for the terrorist network analysis using MCDM problem. First is determining the weight of criteria, which is one of the important factors to be considered for deep impact on the evaluation of the decision. There are two aspects of weight determination; one is a priori weight determined by expert suggestion. This is known as a subjective method. Second is posterior weight determination known as an objective method [44]. It is observed that the limitation in traditional weight determination technique is that only one aspect is not effective or sufficient to determine the weight of criteria because

prior approach determines the weight only one the basis of expert view. So the overall accuracy depends on the expert experience and domain knowledge of the relevant field, which can cause erroneous results if the expert's opinion is getting wrong in some way.

On other hand, using posterior aspect accuracy is depending on the data having after statically analysis. It is well known that all statistical analysis done in some centrality prospects [32]. Each prospect has there on perception to generate results like here we use efficiency centrality [30], betweenness centrality [31], eigenvector centrality [33], and page rank centrality [34]. Each of them has there-on idea to calculate the centrality measures. Here we can see no one centrality measure is dependent on another one in the sense of result determination. So it can be concluded that it is also not sufficient to take only posterior aspect for weight determination, it could also produce less accurate results. A second important factor for terrorist network analysis using MCDM problem is to rank the alternatives as per the given weighted criteria. Many traditional approaches or methods have been proposed for Multi-Criteria Decision Making like AHP [5], Fuzzy AHP [2], TOPSIS [16], Fuzzy TOPSIS [12], Hybrid AHP-TOPSIS [1], SAW [6], DEA [6], EWD [8], Genetic algorithm [4], GDA [9], VIKOR [12], Fuzzy VIKOR [12], Gravity Model [14], EW-TOPSIS [13], PROMETHEE, APLOCO [10] etc. Few of them are used for Terrorist network analysis also. The main issue in these methods was the execution efficiency depends on the number of criteria, number of alternatives their comparison operations and result determination function.

This paper proposed two methods, a two-dimensional approach for criteria weight selection and logarithm based ranking of alternatives to quantify the influential node ability. In criteria weight selection, one dimension is subjective weight selection that uses expert suggestion in fuzzy numbers. Later by applying Extent analysis [37] and Integral value [38] on fuzzy numbers, it determines the criteria weight. Another dimension of criteria weight determination approach is objective, which determine the criteria weight using Aggregated weighting method applied on available data.

The investigation of experimental results shows the proposed model (Fuzzy subjective weight and objective weight by aggregation weighting & Logarithm based ranking) performs well in terms of complexity, accuracy and simplicity. The contribution of this paper is summarised as follows:

Table 1  
Comparison of MCDN methods for terrorist network analysis

Source	Ranking method	Attribute weight selection method	Weighting aspect	Time complexity
William P. Fox and Sean F. Everton [1]	Hybrid AHP & TOPSIS	Expert view-AHP	Priori aspect	$O(mn^2)$ [15, 16]
Li Ze, Sun Duo-Yong, Guo Shu-Quan, Li Bo [2]	FAHP	Expert view-Fuzzy AHP	Priori aspect	$O(mn^2)$ [15]
William P Fox, Brendan Ormond, and Alex Williams [3]	AHP-TOPSIS	Expert view-AHP	Priori aspect	$O(n^2)$ [15]
Saumil Maheshwari and Akhilesh Tiwari [4]	Genetic Algorithm	Degree centrality	Posteriori aspect	$O(V)$ [25]
Pankaj Choudhary, Upasna Singh [5]	AHP	Expert view-AHP	Priori aspect	$O(mn^2)$ [15]
	DEA	Expert view	Priori aspect	$O(wm)$ , where w is the weight of criteria and n is no. of alternative
William P Fox [6]	SAW	Expert view	Priori aspect	$O(mn)$ , where m is no. of alternative and n is no. of criteria
Rithvik Yarlagadda, Diane Felmllee, Dinesh Verma, and Scott Gartner [7]	Centrality measures	No weight calculation	Posteriori aspect	$O(V^3)$ [25]
FanYanga, XiangweiLia, YanqiangXua, XinhuiLiua, JundiWanga, YiZhanga, RuishengZhangb, YabingYao [8]	EWD	EWD	Posteriori aspect	$O(mn)$ , where m is the degree of Vin current G and n is the total number of vertices [8]
Mithun Roy, Indrajit Pan [9]	GDA	Diffusion propagation	Posteriori aspect	$O(\text{Fitness}) * (O(\text{mutation}) + O(\text{crossover})) + O(n \log n)$
Tevfik Bulut [10]	APLOCO	Expert view	Priori aspect	$O(nm)$ , where n is the weight of each criterion i.e. constant and m is no. of alternative
Saurabh Singh, Shashikant Verma, Akhilesh Tiwari [11]	TOPSIS	Expert view	Priori aspect	$O(n^2)$ [16]
Nikolaos Ploskasa, Jason Papathanasiou [12]	Fuzzy TOPSIS	Expert view	Priori aspect	$O(n^2)$ [16]
	VIKOR	Expert view	Priori aspect	$O(n^2)$
Yuanzhi Yang, Lei Yu, Zhongliang Zhou, YouChen, and TianKou [13]	Fuzzy VIKOR	Expert view	Priori aspect	$O(n^2)$
	EW-TOPSIS	Entropy weight	Posteriori aspect	$O(n) + O(n^2)$
Zhe Li, Tao Ren, Xiaqi Ma, Simiao Liu, Yixin Zhang & Tao Zhou [14]	Gravity Model	Gravity Centrality	Not Done	$O(V^2)$ [25]
Saurabh Singh, Shashikant Verma, Akhilesh Tiwari [35]	PROMETHEE	Centrality values	Not done	$O(qn \log n)$ [36]

- Two-dimensional weight determination concepts combine fuzzy-based subjective and objective weighting mechanism to increase accuracy.
- Use logarithmic concept [10] to evaluate the most vital node in the terrorist network.
- Verify the performance of the proposed algorithm in terms of complexity, accuracy and simplicity.

## 2. Related work

In this section, Table 1 shows the deep analysis of all methods used for Terrorist network analysis based on MCDM. Investigation finds the following limitations in existing methods:

- All the methods determine the weight of criteria either prior or posterior mechanism.
- The time complexity of ranking algorithm should be decreased.

- Need a simple method to find the key node in the Terrorist network.

## 3. Model building

In this paper, we proposed a new model of Terrorist Network Mining (TNM) in which a hybrid approach of Fuzzy extent analysis along with Integral value and logarithmic concept of ranking used. The proposed model used three basic steps for identification of a vital node in the terrorist network shown in Fig. 1.

## 4. Methodology

### 4.1. Identification of criteria, terrorist nodes and hierarchical representation of the problem

In the first step, 9/11 world trade centre terrorist attack [47] network investigation problem identified,

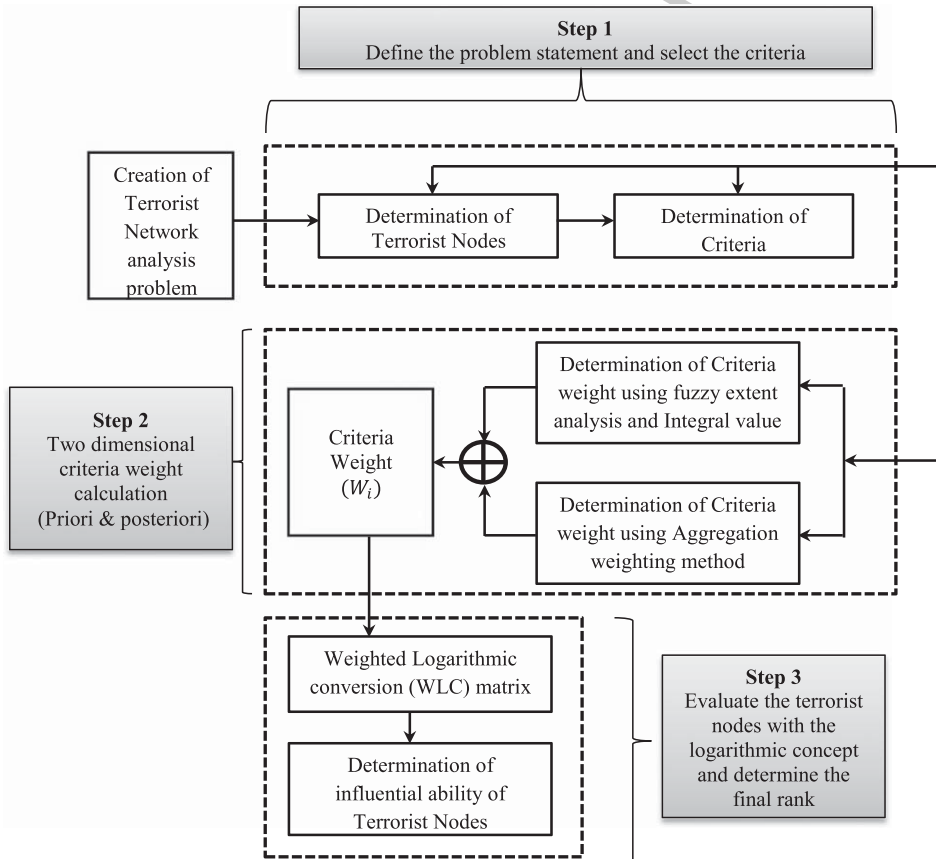


Fig. 1. The proposed model of Terrorist Network Mining.

Table 2  
Description of criteria under consideration

Betweenness centrality [31]	The Betweenness Centrality of node $v$ in a network is defined as: across all node pairs that have the shortest path containing $v$ , the percentage that passes through $v$ .
Closeness centrality [40]	The closeness of a node to the other nodes in a network.
Eigenvector centrality [33]	Leaders of strong cliques are individuals or organizations who are collected to others that are themselves highly connected.
Hub centrality [41]	A node is hub-central to the extent that its out-links are to nodes that have many in-links.
Degree centrality [42]	The number of vertices adjacent to a given vertex in the a-symmetric graph is the degree of that vertex.

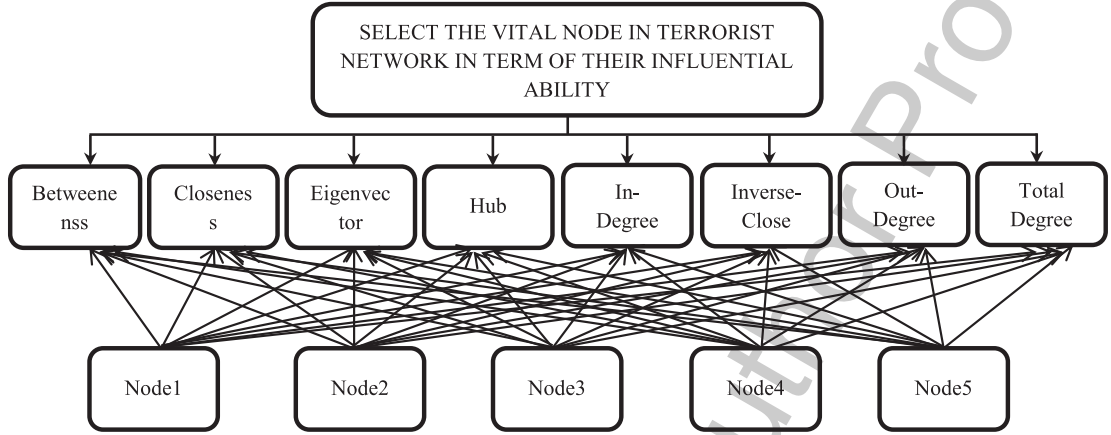


Fig. 2. Hierarchical structure of the criteria-alternative relationship.

in which identification of vital node in term of their influential ability is a most important task considered for investigation. Then the determination of alternatives, i.e. terrorist nodes and criteria has done by using ORA software using world trade centre terrorist attack dataset [48]. First top 8 nodes from 9/11 world trade centre terrorist attack (Fig. 3) were considered as alternatives for evaluation and criteria under consideration are Betweenness centrality, Closeness centrality, Eigenvector centrality, Hub centrality, In-Degree centrality, Inverse-closeness centrality, Out-Degree centrality and Total-Degree centrality. Table 2 gives a short description of the main basic criteria under consideration. Figure 2 shows the hierarchical structure of the criteria-alternative relationship.

#### 4.2. Calculation of criteria weight

The second step is the determination of criteria weight. It is important to calculate the object or efficient weight of all criteria which have been considered for calculating the rank of nodes in a terrorist network.

##### 4.2.1. Basic principal

For that purpose, we need to put our attention on two different dimensions. First is a subjective method, it takes the domain expert suggestion, which is valuable and comes from years of experience in the relevant field and another dimension is Objective method, which calculates the weight of criteria by applying aggregation weighting method in available data generated from ORA software, that would also be valuable or accurate. So, the final weight will be calculated by adding both subjective and objective weights together [45] as shown in Equation 1.

$$W^T = \alpha W^S + \beta W^O \quad (1)$$

where,  $W^T$  is the overall weight of criteria,  $W^S$  is subjective criteria weight,  $W^O$  is objective criteria weight and  $\alpha$  and  $\beta$  are subjective and objective preference information respectively.

Weight calculated by expert suggestion

$$W^S = [a_1, a_2, \dots, a_n] \quad (2)$$

Weight calculated by aggregation weighting method with available data

$$W^O = [b_1, b_2, \dots, b_n] \quad (3)$$

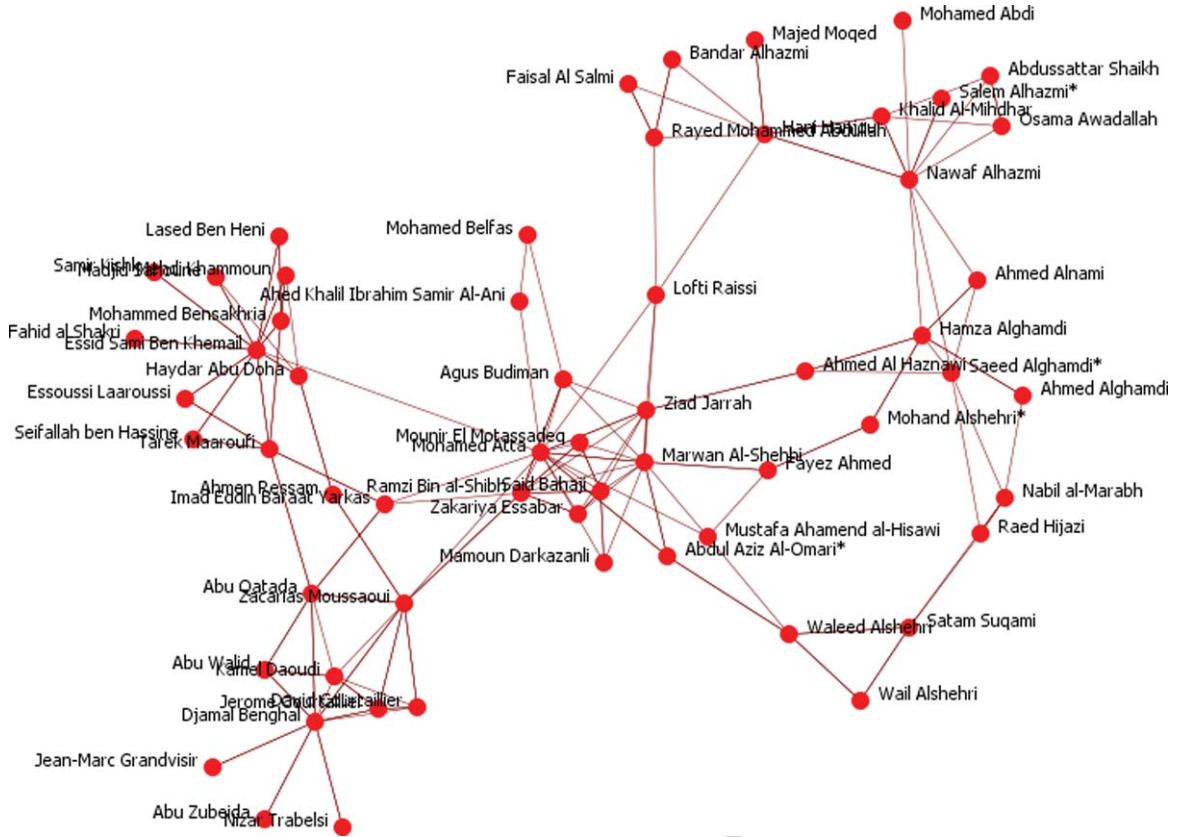


Fig. 3. Social Network graph of the 9/11 world trade centre terrorist attack powered by ORA software.

Therefore,

$$W^T = [\alpha a_1 + \beta b_1, \alpha a_2 + \beta b_2, \dots, \alpha a_n + \beta b_n] \quad (4)$$

As a consequence,

$$W_i^T = \alpha a_i + \beta b_i \quad (5)$$

#### 4.2.2. Weight calculated by expert suggestion

In order to compare the criteria with each other to know the importance of individual one, 9 scale Linguistic term, there corresponding triangular fuzzy number and its reciprocal value given in Table 3.

Criteria compared with each on the basis of suggestions given by experts of relevant field. This comparison shown in Table 4 as follows:

$$a_{ij} = (l_{ij}, m_{ij}, u_{ij}) = a_{ji}^{-1} = \left( \frac{1}{u_{ji}}, \frac{1}{m_{ji}}, \frac{1}{l_{ji}} \right).$$

for  $i, j = 1, 2, \dots, n$  and  $i \neq j$  in matrix  $A$

Table 3  
The Fuzzy scale and its corresponding triangular fuzzy number

Intensity of importance	Triangular fuzzy	Reciprocal	Importance
1	(1,1,1)	(1,1,1)	Equal
3	(2,3,4)	(1/4,1/3,1/2)	Moderate
5	(4,5,6)	(1/6,1/5,1/4)	Strong
7	(6,7,8)	(1/8,1/7,1/6)	Very Strong
9	(9,9,9)	(1/9,1/9,1/9)	Extreme
2	(1,2,3)	(1/3,1/2,1)	Intermediate value
4	(3,4,5)	(1/5,1/4,1/3)	
6	(5,6,7)	(1/7,1/6,1/5)	
8	(7,8,9)	(1/9,1/8,1/7)	

Now Extent analysis method [43] will use to calculate the priority vector of Triangular fuzzy comparison matrix given below.

At first, Fuzzy arithmetic operator will use to sum each row of triangular fuzzy comparison matrix  $A$ .

$$RS_i = \sum_{j=1}^n a_{ij} = \left( \sum_{j=1}^n l_{ij}, \sum_{j=1}^n m_{ij}, \sum_{j=1}^n u_{ij} \right),$$

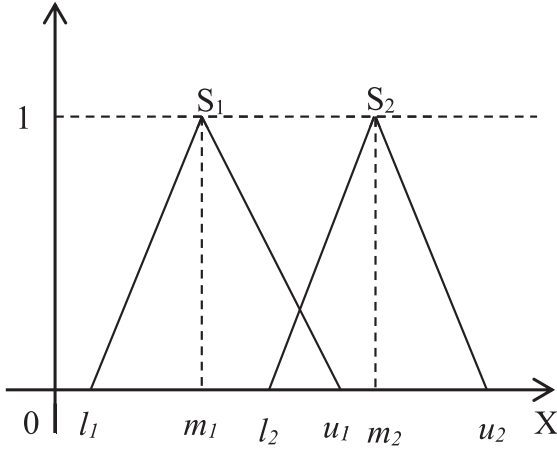


Fig. 4. Triangular Fuzzy graph where,  $l$  is lower,  $m$  is middle, and  $u$  is upper value.

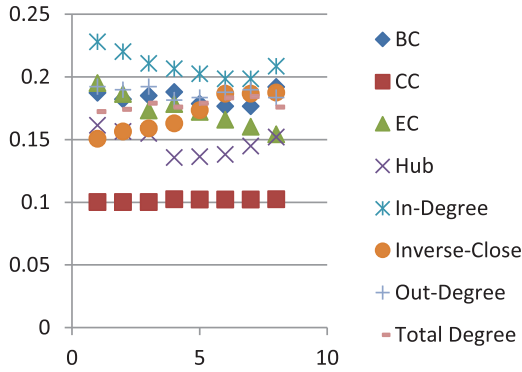


Fig. 5. Line diagram of Criteria value distribution of each alternative in WLC matrix.

$$i = 1, \dots, n \quad (6)$$

Secondly, normalise the summation value of each row of matrix A.

$$S_i = \frac{RS_i}{\sum_{i=1}^n RS_i} = \left( \frac{\sum_{j=1}^n l_{ij}}{\sum_{k=1}^n \sum_{j=1}^n u_{kj}} \right),$$

$$\left( \frac{\sum_{j=1}^n m_{ij}}{\sum_{k=1}^n \sum_{j=1}^n m_{kj}}, \frac{\sum_{j=1}^n u_{ij}}{\sum_{k=1}^n \sum_{j=1}^n l_{kj}} \right),$$

$$i = 1, \dots, n \quad (7)$$

Now in the third step, determine the Synthetic values using Integral value [46] with  $\alpha$  cut level = 0.5 with  $S_i = (l_i, m_i, u_i)$  as follows,

$$I_i = \frac{1}{2} [\alpha u_i + m_i + (1 - \alpha) l_i] \quad (8)$$

Finally, normalised criteria weight for each criterion can calculate by following,

$$W_i^E = \frac{I_i}{\sum_{i=1}^n I_i} \quad (9)$$

Now, triangular fuzzy criteria comparison matrix will generate weights for all criteria under consideration in Table 5.

#### 4.2.3. Weight calculated by aggregation weighting method

Step 1: Create the decision matrix A with  $m \times n$  values where  $m$  is the total number of alternatives and  $n$  is the total number of criteria generated by ORA software.

Step 2: Convert each value of decision matrix in proportional value for the purpose to avoid over dispersion.

$$P_{ij} = \frac{a_{ij}}{\sum_{i=1}^n a_{ij}}, \quad (10)$$

where  $i = 1, \dots, m$  and  $j = 1, 2, \dots, n$

Step 3: Calculate the maximum value from each criteria value.

$$C_j = \max_j (P_{ij}), \quad (11)$$

where  $i = 1, \dots, m$  and  $j = 1, 2, \dots, n$

Step 4: Determine the criteria weight

Table 4  
Triangular Fuzzy comparison matrix A

	Betweenness	Closeness	Eigenvector	Hub centrality	In-Degree	Inverse-closeness	Out-Degree	Total-Degree
Betweenness	(1,1,1)	(1,2,3)	(1,2,3)	(1,2,3)	(1,2,3)	(3,4,5)	(4,5,6)	(5,6,7)
Closeness	(1/3,1/2,1)	(1,1,1)	(1,2,3)	(1,2,3)	(1,2,3)	(2,3,4)	(3,4,5)	(4,5,6)
Eigenvector	(1/3,1/2,1)	(1/3,1/2,1)	(1,1,1)	(1,2,3)	(1,2,3)	(2,3,4)	(3,4,5)	(4,5,6)
Hub centrality	(1/3,1/2,1)	(1/3,1/2,1)	(1/3,1/2,1)	(1,1,1)	(1,2,3)	(1,2,3)	(2,3,4)	(3,4,5)
In-Degree	(1/3,1/2,1)	(1/3,1/2,1)	(1/3,1/2,1)	(1/3,1/2,1)	(1,1,1)	(1,2,3)	(2,3,4)	(3,4,5)
In-close	(1/3,1/2,1)	(1/3,1/2,1)	(1/4,1/3,1/2)	(1/3,1/2,1)	(1/3,1/2,1)	(1,1,1)	(2,3,4)	(3,4,5)
Out-Degree	(1/5,1/4,1/3)	(1/4,1/3,1/2)	(1/5,1/4,1/3)	(1/4,1/3,1/2)	(1/4,1/3,1/2)	(1/4,1/3,1/2)	(1,1,1)	(2,3,4)
Total-Degree	(1/6,1/5,1/4)	(1/5,1/4,1/3)	(1/6,1/5,1/4)	(1/5,1/4,1/3)	(1/5,1/4,1/3)	(1/5,1/4,1/3)	(1/4,1/3,1/2)	(1,1,1)



Table 5  
Subjective Criteria weight of each criterion

Criteria	$RS_I$	$S_I$	$I_I$	$W_i^E$
Betweenness	(17,24,31)	(0.12062, 0.22799, 0.41699)	0.2483975	<b>0.22462</b>
Closeness	(13.33333, 19.5, 26)	(0.09459, 0.18525, 0.34974)	0.2037075	<b>0.18421</b>
Eigenvector	(12.66667, 18, 24)	(0.08987, 0.17099, 0.32283)	0.18867	<b>0.17061</b>
Hub centrality	(9, 13.5, 19)	(0.06386, 0.12825, 0.25558)	0.143985	<b>0.1302</b>
In-Degree	(8.33333, 12, 17)	(0.05913, 0.11399, 0.22868)	0.1289475	<b>0.1166</b>
In-closeness	(7.36667, 9.91667, 13.33333)	(0.05227, 0.09421, 0.17935)	0.10501	<b>0.09496</b>
Out-Degree	(4.31667, 7.41667)	(0.03063, 0.05415, 0.09977)	0.059675	<b>0.05396</b>
Total-Degree		018, 0.04305)	0.02748	<b>0.02485</b>

Table 6  
Criteria-alternative comparison matrix a generated by ORA software

Criteria	BC	CC	EC	Hub	In-degree	Inverse-close	Out-degree	Total degree
Mohamed Atta	0.09	0.021	0.623	0.216	0.25	0.522	0.05	0.151
Marwan Al-Shehhi	0.048	0.021	0.557	0.259	0.2	0.448	0.067	0.134
Ziad Jarrah	0.072	0.021	0.44	0.275	0.133	0.419	0.05	0.092
Ramzi Bin al-Shibh	0.095	0.052	0.489	0.496	0.1	0.374	0.133	0.118
Said Bahaji	0.017	0.051	0.428	0.486	0.067	0.268	0.117	0.092
Zakariya Essabar	0	0.051	0.362	0.463	0.033	0.159	0.083	0.059
Mounir El Motassadeq	0	0.05	0.299	0.381	0.033	0.159	0.067	0.05
Zacarias Moussaoui	0.124	0.052	0.22	0.302	0.117	0.153	0.117	0.118

Table 7  
Objective criteria weight of each criterion

Criteria	$C_j$	$A_j$	$SD_j$	$Z_j$	$W_j^D$
Betweenness	0.278	0.125	0.105	1.454	<b>0.13305</b>
Closeness	0.163	0.125	0.049	0.775	<b>0.07091</b>
Eigenvector	0.182	0.125	0.039	1.478	<b>0.13521</b>
Hub centrality	0.172	0.125	0.039	1.223	<b>0.11183</b>
In-Degree	0.268	0.125	0.083	1.727	<b>0.15802</b>
Inverse-closeness	0.209	0.125	0.059	1.420	<b>0.12994</b>
Out-Degree	0.194	0.125	0.048	1.456	<b>0.13318</b>
Total-Degree	0.186	0.125	0.043	1.398	<b>0.12787</b>

Find the average value of each criterion,

$$A_j = \text{AVERAGE}_j (P_{ij}), \quad (12)$$

where  $i = 1, \dots, m$  and  $j = 1, 2, \dots, n$

Calculate the standard deviation of each criterion,

$$SD_j = \sqrt{\frac{\sum_{i=1}^m (P_{ij} - \bar{P}_{ij})^2}{m-1}}, \quad (13)$$

where  $m$  is the total number of alternatives.

Mean ( $\bar{P}_{ij}$ ) can be calculated by following,

$$\bar{P}_{ij} = \frac{\sum_{i=1}^m P_{ij}}{m}, \quad (14)$$

where  $m$  is the total number of alternatives.

Then normalize the Maximum criteria, Average value and Standard deviation value for each crite-

tion using standardize function (Z-Score formula) as follows,

$$Z_j = \frac{(C_j - A_j)}{SD_j}, \text{ where } j = 1, \dots, n \quad (15)$$

Finally, criteria weight can be calculated as follows,

$$W_j^D = \frac{Z_j}{\sum_{j=1}^n Z_j}, \text{ where } j = 1, \dots, n \quad (16)$$

After applying the Equation number (10)–(16) in matrix A shown in Table 5, resultant values are shown in Table 7.

#### 4.2.4. Two-dimensional final criteria weight calculation

Result of Table 5 represented using Equation (2)

$$W^S = [0.22462, 0.18421, 0.17061, 0.1302, 0.1166, 0.09496, 0.05396, 0.02485]$$

Result of Table 7 represented using Equation (3)

$$W^O = [0.13305, 0.07091, 0.13521, 0.11183, 0.15802, 0.12994, 0.13318, 0.12787]$$

Then,

Result of equation 5 as follows ( $\alpha=0.6$ ,  $\beta=0.4$ ),

Table 8  
Final Two dimensional Criteria weight of each criterion

Weight	BC	CC	EC	Hub	In-Degree	Inverse-Close	Out-Degree	Total Degree
$W^T$	0.188	0.139	0.156	0.123	0.133	0.109	0.086	0.066

Table 9  
The SPC values

	BC	CC	EC	Hub	In-degree	verse-close	Out-degree	Total degree
Min & Max	Max	Max	Max	Min	Max	Min	Min	Min
Mohamed Atta	0.034	0.031	0	0	0	0.369	0	0.101
Marwan Al-Shehhi	0.076	0.031	0.066	0.043	0.05	0.295	0.017	0.084
Ziad Jarrah	0.052	0.031	0.183	0.059	0.117	0.266	0	0.042
Ramzi Bin al-Shibh	0.029	0	0.134	0.28	0.15	0.221	0.083	0.068
Said Bahaji	0.107	0.001	0.195	0.27	0.183	0.115	0.067	0.042
Zakariya Essabar	0.124	0.001	0.261	0.247	0.217	0.006	0.033	0.009
Mounir El Motassadeq	0.124	0.002	0.324	0.165	0.217	0.006	0.017	0
Zacarias Moussaoui	0	0	0.403	0.086	0.133	0	0.067	0.068

Table 10  
The LC matrix

	BC	CC	EC	Hub	In-degree	Inverse-close	Out-degree	Total degree
Mohamed Atta	1.408296	1.41123	1.442545	1.442545	1.4425455	1.15934319	1.44254546	1.346818749
Marwan Al-Shehhi	1.36889	1.41123	1.378	1.399593	1.3929241	1.203631617	1.42514292	1.361720177
Ziad Jarrah	1.391034	1.41123	1.280769	1.384475	1.3331951	1.222342982	1.44254546	1.400552867
Ramzi Bin al-Shibh	1.413195	1.442545	1.319128	1.213208	1.3062552	1.253069568	1.36261084	1.376164528
Said Bahaji	1.341665	1.441506	1.271838	1.219713	1.2807688	1.334877367	1.37708123	1.400552867
Zakariya Essabar	1.327353	1.441506	1.225653	1.235056	1.2559067	1.436338169	1.40927224	1.433261413
Mounir El Motassadeq	1.327353	1.440468	1.185709	1.294498	1.2559067	1.36338169	1.42514292	1.442545456
Zacarias Moussaoui	1.442545	1.442545	1.140499	1.359944	1.3199438	1.442545456	1.37708123	1.376164528

Table 11  
The WLC matrix

	BC	CC	EC	Hub	In-degree	Inverse-close	Out-degree	Total degree
Mohamed Atta	0.26476	0.196161	0.225037	0.177433	0.1918585	0.126368408	0.12405891	0.088890037
Marwan Al-Shehhi	0.257351	0.196161	0.214968	0.17215	0.1852589	0.131195846	0.12256229	0.089873532
Ziad Jarrah	0.261514	0.196161	0.1998	0.17029	0.1773149	0.133235385	0.12405891	0.092436489
Ramzi Bin al-Shibh	0.265681	0.200514	0.205784	0.149225	0.1737319	0.136584583	0.11718453	0.090826859
Said Bahaji	0.252233	0.200369	0.198407	0.150025	0.1703423	0.0501633	0.11842899	0.092436489
Zakariya Essabar	0.249542	0.200369	0.191202	0.151912	0.1670356	0.656086	0.12119741	0.094595253
Mounir El Motassadeq	0.249542	0.200225	0.184971	0.159223	0.1670356	0.15656086	0.12256229	0.095208
Zacarias Moussaoui	0.271199	0.200514	0.177918	0.167273	0.1755525	0.157237455	0.11842899	0.090826859

Table 12  
The  $\theta$  score of the terrorist nodes

	$\alpha_i$	$\beta_i$	$\theta_i$
Mohamed Atta	1.394566717	1.442545456	0.966740224
Marwan Al-Shehhi	1.369520795	1.442545456	0.949377914
Ziad Jarrah	1.354811558	1.442545456	0.93918119
Ramzi Bin al-Shibh	1.339530895	1.442545456	0.928588343
Said Bahaji	1.327743222	1.442545456	0.920416903
Zakariya Essabar	1.332414463	1.442545456	0.923655097
Mounir El Motassadeq	1.335327969	1.442545456	0.925674795
Zacarias Moussaoui	1.358949169	1.442545456	0.942049461

Table 13  
Rank of the terrorist nodes

	$\theta$ Score	Rank
Mohamed Atta	0.966740224	1
Marwan Al-Shehhi	0.949377914	2
Ziad Jarrah	0.93918119	4
Ramzi Bin al-Shibh	0.928588343	5
Said Bahaji	0.920416903	8
Zakariya Essabar	0.923655097	7
Mounir El Motassadeq	0.925674795	6
Zacarias Moussaoui	0.942049461	3

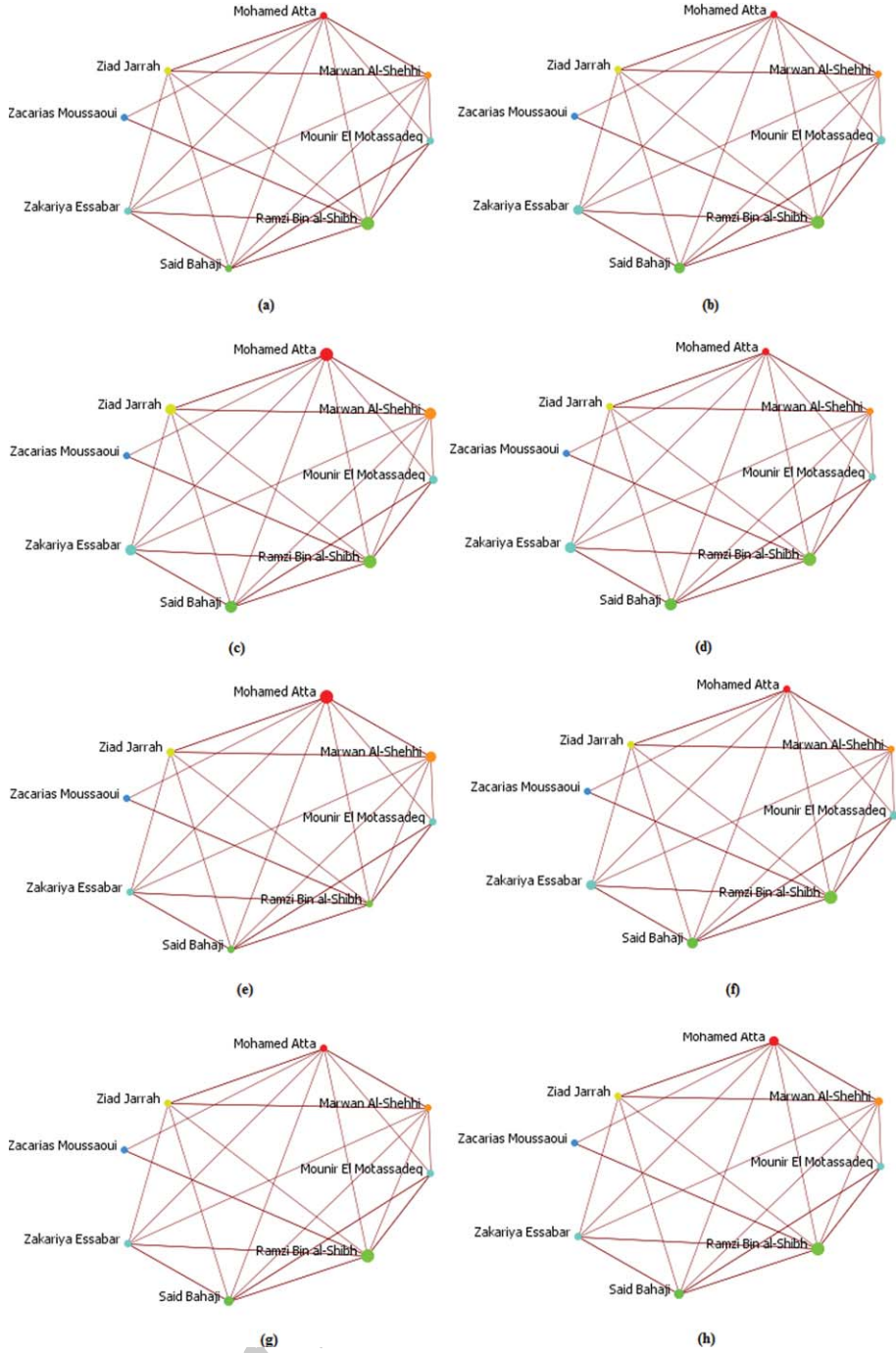


Fig. 6. Rank of top 8 nodes in 9/11 world trade center terrorist attack based on (a) Betweenness (b) Closeness (c) Eigenvector (d) Hub (e) In-degree (f) Inverse-closeness (g) Out-degree (h) Total degree centrality values.

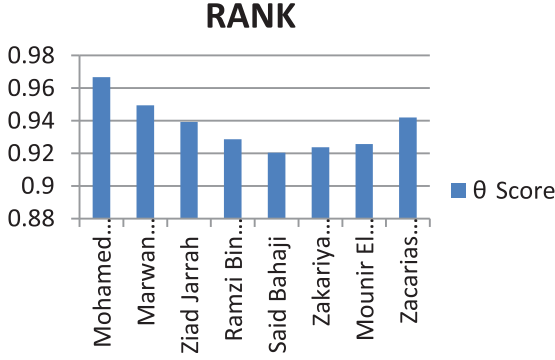


Fig. 7. Rank according to  $\theta$  score of top 8 node of 9/11 world trade center terrorist attack.

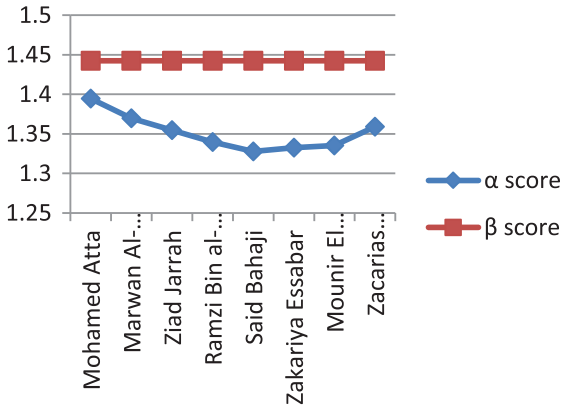


Fig. 8. Distance between  $\alpha$  score and  $\beta$  score.

## 5. Evaluate the terrorist nodes with the logarithmic concept and determine the final rank

### Step 1: Formation of the Decision Matrix

Table 6 considered a decision matrix  $A$  for our evaluation.

### Step 2: Determine the Starting Point Criteria (SPC) value

- Investigate, a criteria value is required to be either maximum or minimum by using criteria weight value calculated earlier.
- Find  $\max_j a_{ij}$ , where  $i = 1, \dots, m$  and  $j = 1, \dots, n$  in matrix  $A$ , if criteria required being maximum.
- Find  $\min_j a_{ij}$ , where  $i = 1, \dots, m$  and  $j = 1, \dots, n$  in matrix  $A$ , if criteria required being minimum.
- Now create an SPC value matrix by the following equation,

$$SPC_{ij} =$$

$$\begin{cases} \max_j a_{ij} - a_{ij}, & \text{if } SPC_{ij} \text{ is maximum starting point criteria} \\ a_{ij} - \min_j a_{ij}, & \text{if } SPC_{ij} \text{ is minimum starting point criteria} \end{cases} \quad (17)$$

where,  $i = 1, \dots, m$  and  $j = 1, \dots, n$

### Step 3: Devise the Logarithmic conversion (LC) matrix

$$L_{ij} = \frac{1}{\ln(SPC_{ij} + 2)}, \quad (18)$$

where  $i = 1, \dots, m$  and  $j = 1, \dots, n$

Here +2 added in each row of  $SPC_{ij}$  matrix values. Now LC value will be calculated by taking the natural log of the multiplicative inverse of resultant value.

### Step 4: Determine the Weighted Logarithmic Conversion (WLC) matrix

In this step, matrix value will determine by multiplying the weight coefficient value ( $W^T$ ). with LC matrix ( $L_{ij}$ ) values.

$$WLC_{ij} = W_j^T * L_{ij} \quad (19)$$

where  $i = 1, \dots, m$  and  $j = 1, \dots, n$

### Step 5: Determination of influential ability of Terrorist Nodes by evaluating the best alternative

$$\alpha_i = \sum_{j=1}^n WLC_{ij}, \text{ where } i = 1, \dots, m \quad (20)$$

Table 14  
Rank comparison of existing methods with proposed method

Terrorist Name	Proposed Model	AHP	TOPSIS	PROMETHEE	ORA
Mohamed Atta	<b>0.966740224</b>	<b>0.16746199</b>	<b>0.694784</b>	<b>0.072509601</b>	<b>0.81</b>
Marwan Al-Shehhi	0.949377914	0.142320614	0.550998	0.04150439	0.67
Ziad Jarrah	0.93918119	0.122712178	0.572669	0.003987487	0.454
Ramzi Bin al-Shibh	0.928588343	0.159309006	0.652946	0.049604264	0.374
Said Bahaji	0.920416903	0.114722458	0.368647	-0.00097713	0.268
Zakariya Essabar	0.923655097	0.082923163	0.327491	-0.049024867	0.159
Mounir El Motassadeq	0.925674795	0.072493328	0.323254	-0.072907217	0.159
Zacarias Moussaoui	0.942049461	0.138067263	0.660209	-0.044696529	0.153

$$X_j = \max_j WLC_{ij} \quad (21)$$

where  $i = 1, \dots, m$  and  $j = 1, \dots, n$

$$\beta_{i=1, \dots, m} = \sum_{j=1}^n X_j \quad (22)$$

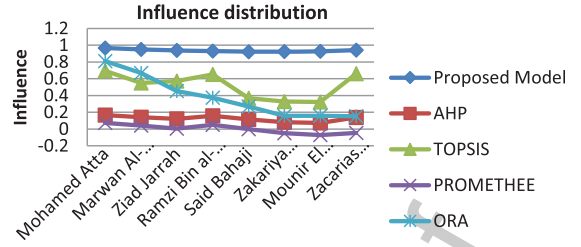
$$\theta_i = \frac{\alpha_i}{\beta_i} \quad (23)$$

where  $i = 1, \dots, m$  and  $0 \leq \theta_i \leq 1$

Here,  $\theta_i$  is the final score of each terrorist node which shows the influential ability of each individual. Difference between  $\beta_i$  and  $\alpha_i$  is the sum of the optimal solution and the score of  $\alpha_i$ . which is closer to  $\beta_i$  score determined as the most influential terrorist node among all other nodes.

## 6. Result analysis & discussion

Besides identification of the most important node in term of its influential capability in a given terrorist social network s become a strategic issue for law enforcement agencies and retain a preventive



Top 8 nodes in 9/11 world trade center terrorist attack

Fig. 9. Influence distribution of Top 8 nodes in 9/11 world trade center terrorist attack.

advantage in global terrorism. This study proposes a Logarithm concept based vital node identification technique in terrorist node, the result shown in Table 13. The proposed approach also combines the fuzzy-based subjective criteria weight determination and aggregated criteria weight determination technique uses an objective method to calculate two dimensional more accurate criteria weight. Finally, a case study of the 9/11 world trade centre terrorist attack in which 60 terrorists were involved is used to illustrate the operational procedure of the proposed approach. Figure 6 shows the graph of top 8

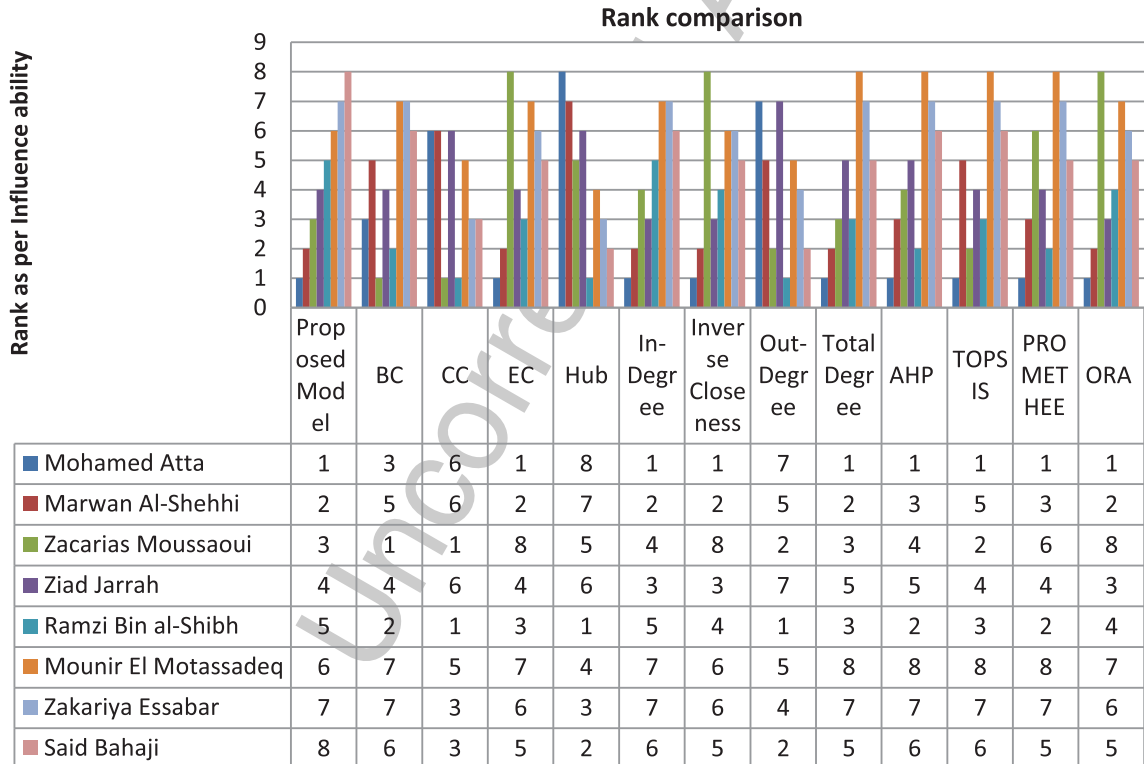


Fig. 10. Rank comparison of existing methods with proposed method.

Table 15  
Time complexity comparison of various methods and proposed methods

	Proposed model	AHP	TOPSIS	PROMETHEE	VIKOR	BC	CC	EC	Degree
T	$O(mn)$	$O(mn^2)$	$O(n^2)$	$O(qn \log n)$	$O(n^2)$	$O(VE)$	$O(m^3)$	$O(m^3)$	$O(m)$

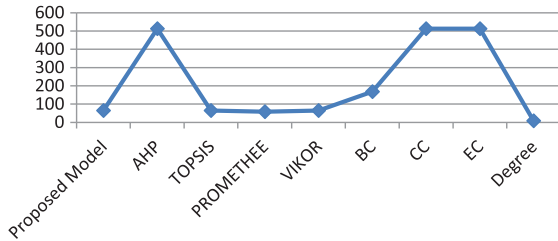


Fig. 11. Time complexity distribution of used dataset, where  $m = 8$ ,  $n = 8$  and total edges are 21 (Fig. 6).

nodes from 9/11 world trade centre terrorist attack network data which are selected by ORA software. Here node size is represented as per there centrality values. Rank identified according to the  $\theta$  score from Table 4, shown in Fig. 7. Distance between  $\alpha$  score and  $\beta$  score, shown in Fig. 8. A node which has a closer distance of  $\alpha$  score to  $\beta$  score will have higher influential ability and will be the most vital node in the terrorist network.

## 7. Comparison with existing methods

In this section, we compare the proposed method with few existing methods like for most centralized node identification in 9/11 terrorist attack dataset. Table 14 shows the influences by different methods which are shown in graphical representation in Fig. 9.

Figure 10 compares the ranking order of top 8 nodes from 9/11 dataset generated by different existing methods and propose model.

Table 13 shows the time complexity of each existing methods and proposed method.

## 8. Conclusion

A significant problem with the researchers of terrorist network analysis is to choose a suitable method which should have a multi prospective criterion to identify a vital node in a terrorist network. In this paper, we compared the results determined by the proposed method with different traditional methods. The result generated by other methods also produces the same result for the top most influential node in

9/11 terrorist attack data, in comparison to the proposed model. It concludes that the proposed model generated an optimal solution.

We focused first on the criteria of weight selection methods. Fuzzy logic based subjective criteria weight calculated and added with the proposed aggregated weighting method to produce more accurate criteria weight known as two-dimensional weight calculations later theses weights used in logarithmic concept to determine most influential node in 9/11 terrorist attack dataset.

Finally, extensive comparison with different methods done to study the influence distribution in the selected application area. It has also observed that the time complexity of the proposed model is less in comparison to other traditional methods. In future work, we plan to incorporate genetic clustering concept and study the effect. We also plan to use fuzzy logic in logarithmic concept to overcome uncertainty.

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