

# INTERNATIONAL CONFERENCE ON ADVANCED TECHNOLOGIES, COMPUTER ENGINEERING AND SCIENCE

Alanya, TURKEY

26-28  
APRIL  
2019



**2019**  
**PROCEEDING BOOK**

# ICATCES 2019 Proceeding Book

International Conference on Advanced Technologies,  
Computer Engineering and Science

26-28 Apr 2019 / Alanya, Turkey

# Node Weighting Method in Centrality Measure of Complex Networks

O. FINDIK<sup>1</sup> and E. ÖZKAYNAK<sup>1</sup>

<sup>1</sup>Karabuk University, Karabuk/Turkey, [oguzfindik@karabuk.edu.tr](mailto:oguzfindik@karabuk.edu.tr)

<sup>1</sup>Karabuk University, Karabuk/Turkey, [eozykaynak@karabuk.edu.tr](mailto:eozykaynak@karabuk.edu.tr)

**Abstract**—Complex networks are one of the tools commonly used in modeling all kinds of events that are related to each other. The identification of effective nodes in complex networks is an important issue needed for the analysis of complex networks. Degree, closeness and betweenness measures are the most important centrality measures commonly used to analyze networks. As a local metric, degree is relatively simple and less effective, although global measures such as the measure of closeness and betweenness can better define effective nodes. However, there are still some disadvantages and limitations of all of these measures. Degree, closeness and betweenness measures are only the data obtained from the topological structure of the network. However, in determining the centralization of the node, other than the topological structure of the network that affect the formation of the network, but not expressed in the topological structure of the network is also effective. In this study, the node weighting method was developed for the determination of node centralization in the network and compared with the current node centering criteria. The experimental study was conducted on a network of players and played competitions in the Australian Open Tennis Tournament held between 2000-2017. By using criteria such as time, experience and success, the weights of the nodes were calculated and compared with the node centering criteria. The results obtained from the experimental study show that the node weighting method gives successful results in the determination of active nodes in complex networks.

**Keywords** - Complex Networks, Node Weighting, Data Mining

## I. INTRODUCTION

The communication and emerging interaction process of people with each other has been developing and growing for centuries. Together with technological developments, these communications and sharing methods are also developing. Everything that people do together is a natural result of communication and sharing. The fact that people connect and interact with each other directly or indirectly constitutes the elements of a complex network. Structures defined as complex networks are not limited to expressing the relationship between people and each other. Everything that has interactions or connections in various ways is actually a part of complex networks. Complex network science, gather and analyze all kinds of structure, system and situation that have the relationship, connection and sharing between them, direct and indirect ways, within the framework of certain rules and disciplines. [1]. In the analysis of complex networks, the position and importance of the nodes in the network is important to examine how the information flows in the network and the structure of the network [2]. Centrality is one of the

most important concepts that are considered when examining the structure of a complex network. The concept of centrality is a set of indicators that reveal the importance of the node in the network in a complex network. In most complex networks, some links or nodes are more central than others. To measure the centrality of the nodes, there are measures of centrality, such as degrees, interactivity, proximity and eigenvectors. These measures of centrality are obtained by using the topological information of the network. However, in complex networks, only topological consideration of the centrality of a node may not be sufficient to reveal some important nodes in the network. Increasing or decreasing the effect of the nodes on the neighbors with the expansion of the network over time can be important in determining the node center. According to the topological structure, the metrics that determine the node centrality do not take into account the factors that may affect the node centrality except the topological structure of the network. Depending on the time and the different conditions, changes in the nodes state in the network should be taken into account in revealing the node centrality. In this study, which emphasizes the importance of node weighting in uncovering node centrality, in order to calculate the weights of the nodes, the common criteria for all nodes in the network were determined and the weights of the nodes were calculated using the multi-criteria decision-making method. In the experimental study, node weighting method was used to reveal the node centrality in the network in a network composed of competitions played between 2000-2017 Australia in the Open Tennis Tournament. The experimental results show that the node weighting method gives successful results in order to reveal the node centrality.

## II. METHOD

### A. Network Centrality

The centrality factor is an important concept, particularly in complex network structures. Network centrality can be defined as the criterion of nodes in a complex network. [3]. This criterion measures the strength of a node's dependence on other nodes while on the other hand it measures its effectiveness on other nodes. In determining the important and active locations in the network, there are usually the most important or most known nodes. Different centrality measures have been proposed to determine network centrality. The proposed centrality criteria and the characteristics of the node position in social networks have been tried to be defined and measured. [4].

### *Centrality Criteria*

A complex network analysis is the study of the interactions that are in interaction with each other, the types of relationships and the interaction between relationships. The concept of relationship can be defined as the link between social entities. [4]. The individuals, objects and communities that form the basis of relations are expressed by nodes in network analysis [5]. The relations established in the social networks due to the interaction between the nodes are the equivalent of the concept of links. The relationship between individuals can be expressed in different types such as friendship bond, kinship bond, competition tie, financial tie, belief tie [6-11]. The proposed criteria for examining the link structures between nodes in the networks formed contain information about the importance of the node in the network.

### *Degree Centrality*

Degree centrality, which is the simplest of the criteria used to reveal network centrality, is calculated by the number of links to a node on the network. Although it is simple to calculate, it is an important criterion that can indicate the position of the node in the network. In most complex networks, the more links a node has, the more important and powerful it is. In fact, the node with the highest degree can be interpreted as the most active member of the network. In cases where links are direct, the number of in-degree links and the number of out-degree links are calculated separately [12,13].

### *Eigenvector Centrality*

The Eigenvector Centrality criterion, another criterion used to elucidate network-centrality, is a criterion that indicates that the links owned by the node taken into account when calculating the rating are not equal. For a node on the network, the effect of links to key nodes may be more than any other ordinary links. The fact that the nodes on which it is connected are more centrally indicate that the node will be in a more central position. When calculating this criterion, the totality of the neighbors' centrality is taken into account. [14,15].

### *Betweenness Centrality*

Betweenness centrality is one of the most complex criteria to be calculated in the centrality criteria, which is one of the criteria used to elucidate network-centrality. Betweenness centrality is found by the ratio of the most shortcuts that pass through a node in the network. First, the most shortcuts be found between all node pairs in the network. Ratio between this shortcuts (geodesics) and number of path that node is located on gives the betweenness centrality. Because it is a very costly measure that can be calculated on large networks, it can also be calculated by going down to certain level neighbors. Due to their location, the nodes with a high degree of gravity are more important than the other nodes and will be more aware of what is going on in the network [12,13].

### *Closeness Centrality*

The Closeness centrality is another centrality criterion used to reveal network. It is found that any node in the network obtains the shortest mean distance to all other nodes (geodesic distance). If the links are directional, these aspects should be taken into consideration when finding the shortcuts. In this case, two different metric which are in-closeness and out-closeness are calculated [12, 14].

### *B. Gephi*

Because of the large number of nodes and links due to the nature of complex networks, it is important to use appropriate analysis tools to ensure that the data obtained from these nodes and links are accurate, understandable and interpretable. In complex network analysis, it is important to detect network centrality, to find shortest paths between nodes, to obtain clustering coefficients and to calculate degree distributions. In addition, the visualization of the data obtained in an appropriate manner has an important place in the understanding and analysis of the structure of the network. In this study, Gephi software was used to obtain data showing the centrality of the network of tennis competitions and to visualize the network. Gephi software is preferred because of it is free and has been instrumental in choosing to provide detailed options during the performance of the analysis data and the visualization of the network[18].

### *C. Approach of Logarithmic Concept (APLOCO)*

One of the multi-criteria decision-making methods, APLOCO, uses Multi-Layer Perceptron, which is one of the Deep Learning methods in the field of artificial intelligence in determining the weight of criteria. One of the important features of APLOCO is that it is not dependent on artificial intelligence at determining the weight of the criteria. Criterion weights can also be determined by any method. As a result, the determined weights can be used in this method if the criteria weights are determined by any method. The reason for using APLOCO method in this study is that the data used for experimental study does not show normal distribution and allows to calculate the node weight from different criteria. The fact that artificial intelligence does not exclude the data and have the ability to learn from the data constitutes one of the main reasons for using the criteria in the process of determining the weight [16].

### *D. Implementation Steps of APLOCO*

The implementation of APLOCO is completed in 5 steps;

- Step 1: Building the decision matrix (DM)
- Step 2: Calculation of starting point criteria (SPC) values
- Step 3: Forming the logarithmic conversion (LC) matrix
- Step 4: Determining the weights of criteria (WC) and calculating the weighted logarithmic conversion (WLC) matrix
- Step 5: Determination of the best alternative (BA)

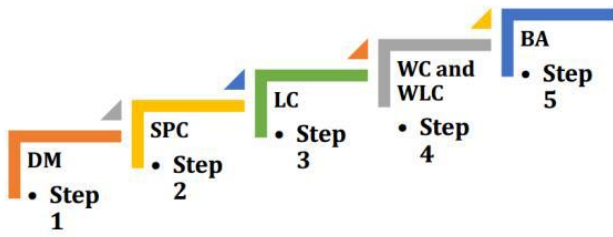


Figure 1: APLOCO Application Steps.

### Step 1: Building the decision matrix (DM)

The generated decision matrix ( $X_{ij}$ ) is a  $C \times R$  dimensional matrix that contains the criteria in the rows and the alternatives in the columns. Here, respectively, the number of criteria and the number of alternatives are re-evaluated.  $X_j$  refers to decision variables and  $X_i$  refers to the values of decision variables. This matrix is shown in the equation in (1).

$$X_{ij} = \begin{matrix} \text{Criteria} \\ \begin{bmatrix} C_1 \\ C_2 \\ \dots \\ C_c \end{bmatrix} \end{matrix} \begin{matrix} \text{Alternatives} \\ \begin{bmatrix} A_1 & A_2 & \dots & A_r \end{bmatrix} \end{matrix} = \begin{bmatrix} X_{11} & X_{12} & \dots & X_{1r} \\ X_{21} & X_{22} & \dots & X_{2r} \\ \dots & \dots & \dots & \dots \\ X_{c1} & X_{c2} & \dots & X_{cr} \end{bmatrix} \quad (1)$$

### Step 2: Calculation of starting point criteria (SPC) values

At this stage, if the criterion value needs to be maximum, the maximum value between the values of the corresponding criterion in that line is determined as the maximum value. If the criterion value is to be minimum, the minimum value between the values in the corresponding row is set to the minimum value. In this case, if the desired criteria is the maximum, the criteria values in the row to which the maximum value belongs are subtracted from the maximum value. On the other hand, if the required criterion is minimum, the minimum value is subtracted from the criterion values in which it belongs. In order to perform said operations, the  $P_{ij}$  values containing the maximum and minimum starting point criterion values are obtained using the equations (2), (3) and (4). Here,  $i = 1, 1 \dots c$ ;  $j = 1, \dots, r$ .

$$P_{ij} = \begin{cases} \max x_{ij} - x_{ij} & \text{if } P_{ij} \text{ is the maximum starting point criterion.} \\ x_{ij} - \min x_{ij} & \text{if } P_{ij} \text{ is the minimum starting point criterion.} \end{cases} \quad (2)$$

(3)

$$X_{ij} = \begin{bmatrix} X_{11} - \min x_{1j} & X_{12} - \min x_{1j} & \dots & X_{1r} - \min x_{1j} \\ X_{21} - \min x_{2j} & X_{22} - \min x_{2j} & \dots & X_{2r} - \min x_{2j} \\ \dots & \dots & \dots & \dots \\ X_{c1} - \min x_{cj} & X_{c2} - \min x_{cj} & \dots & X_{cr} - \min x_{cj} \end{bmatrix} = \begin{bmatrix} X_{11} & X_{12} & \dots & X_{1r} \\ X_{21} & X_{22} & \dots & X_{2r} \\ \dots & \dots & \dots & \dots \\ X_{c1} & X_{c2} & \dots & X_{cr} \end{bmatrix} \quad (4)$$

### Step 3: Forming the logarithmic conversion (LC) matrix

At this stage, +2 integer value is added to each of the  $P_{11}$ ,  $P_{12}$ ,  $P_{13}$ ,  $\dots$ ,  $P_{1r}$  values in the rows. The LC values are then calculated by calculating the cyclic natural logarithm as opposed to the results obtained. This is done by the equation (5) and the normalization process is completed. Logarithmic transformation matrix is then obtained in equation (6). Here,  $\ln$  is undefined when 0 and negative values occur and  $\ln = 0$ . For

these reasons, +2, greater than 1 in the equation (4), is considered to be an integer value. Another reason to add number 2 as an integer value to the logarithm value at that time is to avoid excessive values and negative values and to ensure that the values are positive.

$$\ln(x) = \log_e(x) \text{ and } L_{ij} = \frac{1}{\ln(p_{ij} + 2)} \text{ for } (i = 1 \dots c \text{ and } j = 1 \dots r) \quad (5)$$

$$X_{ij} = \begin{bmatrix} \frac{1}{\ln(p_{11} + 2)} & \frac{1}{\ln(p_{12} + 2)} & \dots & \frac{1}{\ln(p_{1r} + 2)} \\ \frac{1}{\ln(p_{21} + 2)} & \frac{1}{\ln(p_{22} + 2)} & \dots & \frac{1}{\ln(p_{2r} + 2)} \\ \dots & \dots & \dots & \dots \\ \frac{1}{\ln(p_{c1} + 2)} & \frac{1}{\ln(p_{c2} + 2)} & \dots & \frac{1}{\ln(p_{cr} + 2)} \end{bmatrix} = \begin{bmatrix} l_{11} & l_{12} & \dots & l_{1r} \\ l_{21} & l_{22} & \dots & l_{2r} \\ \dots & \dots & \dots & \dots \\ l_{c1} & l_{c2} & \dots & l_{cr} \end{bmatrix} \quad (6)$$

### Step 4: Determining the weights of criteria (WC) and calculating the weighted logarithmic conversion (WLC) matrix

$W$  is a weight coefficient. Here,

$w_1 \in \mathbb{R} \text{ ve } \sum_{i=1}^n w_i w_1 \in \mathbb{R} \text{ ve } \sum_{i=1}^n w_i = 1$  The equation of the WLC matrix (7) is obtained by multiplying the logarithmic transformed  $l_{ij}$  values by the weight levels ( $w_i$ ) of the criteria determined by any method.

$$T_{ij} = \begin{bmatrix} l_{11}w_1 & l_{12}w_1 & \dots & l_{1r}w_1 \\ l_{21}w_2 & l_{22}w_2 & \dots & l_{2r}w_2 \\ \dots & \dots & \dots & \dots \\ l_{c1}w_n & l_{c2}w_n & \dots & l_{cr}w_n \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} & \dots & t_{1r} \\ t_{21} & t_{22} & \dots & t_{2r} \\ \dots & \dots & \dots & \dots \\ t_{c1} & t_{c2} & \dots & t_{cr} \end{bmatrix} \quad (7)$$

### Step 5: Determination of the best alternative (BA)

The maximum values of the criteria in each order are determined as optimal solution values ( $\beta_j$ ) and after the total score is obtained as  $\beta_{sj}$ . This process is done by the equations (8) and (9). The final scores ( $j_i$ ) of each alternative are calculated by proportioning the total scores ( $\alpha_{si}$ ) of the criterion values of the alternatives to the optimum solution values ( $\beta_{sj}$ ) collected. This process is done by (10) and (11) equations, respectively. The scores obtained from Equation (11) are between 0 and 1, and the scores are allowed to be evaluated within this range. Then the  $\theta_i$  values are sorted from large to small and the first order alternative is considered the most suitable alternative [17].

$$B_{sj} = \{ \max t_{ij} \} \text{ and } B_{ij} = \{ t_{11}, t_{12}, t_{13}, \dots, t_{1n} \} \quad (8)$$

$$B_{si} = \sum_{j=1}^n (l_1, l_2, l_3, \dots, l_n) \quad (9)$$

$$a_{sj} = \sum_{j=1}^n (t_1, t_2, t_3, \dots, t_n) \quad (10)$$



$$0 \leq \theta_i \leq 1 \text{ and } j = 1, 2, \dots, r. \theta_i = \frac{\alpha_i}{\beta_i} \quad (11)$$

### E. Node Weighting Method

Node weighting is the process of calculating the weights of the nodes in the network according to the determined criteria by APLOCO method which is one of the multi criteria decision making methods. In this approach, the calculated node weights indicate the importance of the node in the network. As the weight is reduced, the position of the nodes is away from the centrality of the network as the weight of the node is at the center of the network.

## III. EXPERIMENTAL STUDY

In the experimental study, the network in Figure 2 was formed using the athletes who competed in the Australian Open [17] tennis tournaments between 2000-2017 and the competitions taking place between them.

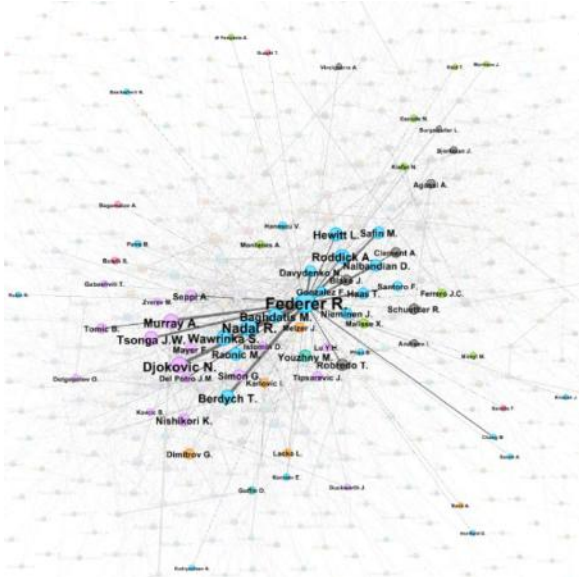


Figure 2: Network Created for Australian Open Tennis Tournament

In the created network, the nodes are composed of athletes and the links are composed of the matches between athletes. The links between the nodes are constructed as unidirectional. The node and link information for the created networks is shown in Table 1.

Table 1: Nodes and Links for the Tennis Network

	Australian Open
Nodes	553
Edges	2156

Centrality measures were calculated by modeling the network through the Gephi program to reveal the nodes in the center of the network.

### A. Criteria

APLOCO method was used for the proposed node weighting method and node weights were calculated. For data to be used in APLOCO, time, experience, success and tour criteria are determined according to the structure and characteristics of the network. The criteria in the generated network are shown in (12), (13), (14) and (15).

#### Time Criteria

In time-dependent expanding networks, the position of the nodes over time is an effective element in determining their position in the network. In the tennis network, the effectiveness of an athlete who has decreased or ended his / her participation in tennis competitions must decrease in the network. Therefore, the time criterion should be used as an important parameter in determining the effectiveness and importance of nodes in the network. The time criterion used in this study was calculated as shown in equation (12) as the ratio of the last year of the tournament year played to the time interval of the netting weighted network.

$$Time = \frac{\text{The Last Competition Year} - \text{The First Tournament Date}}{\text{The Last Year Calculated} - \text{The First Year Calculated} + 1} \quad (12)$$

Here, the last competition year refers to the year of the last tournament where the athlete participated, the first competition year refers to the year of the first tournament where the athlete participated, the last year calculated refers the date of the last tournament in the network that will be weighted, the calculated year refers to the date of the first tournament that took place in the network to be weighted. A value of 1 in Equation 2.30 is used to avoid the result of 0 when the desired range corresponds to the same year.

#### Experience Criteria

In a network of sporting events, such as a tennis tournament, the athlete's experience in the tournament is effective being in the center of the network. The more matches an athlete has, the more likely he is to link with so many athletes. The criterion of experience used in this study was calculated as shown in (13) as the ratio of the number of matches played in tournaments to the total number of matches in tournaments.

$$Experience = \frac{\text{The Number Of Matches Played}}{\text{The Total Number Of Matches}} \quad (13)$$

The number of matches played here is the total number of matches of the athlete, while the total number of matches played represents the total number of competitions in tournaments.

#### Success Criteria

The expansion of the network in sports tournament networks is not just about joining new athletes to the network. Athletes with high success in the network also have effects on the expansion of the network. It is possible for the athlete with high success to be positioned in the center of the network due to the fact that he competes with many athletes. The success in the tournaments as well as the number of matches played by the athlete is a criterion that increases the effectiveness and importance of the network. The success criterion used in this study was calculated as the ratio of the number of matches won by the athlete in the tournaments to the total number of matches played in the tournaments as shown in the equality (14).

$$Success = \frac{\text{The Number Of Matches Won}}{\text{The Number Of Matches Played}} \quad (14)$$

Here, the number of matches won by the athlete refers to matches won by athlete in the matches played, the number of matches played refers to the total number of competitions the athlete has in tournaments.

#### Tour Criteria

Considering the fact that the tennis tournaments are composed of tours within themselves, as in most sports events, the number of rounds played by an athlete in each tournament is one of the factors in the position of the athlete in the network. The fact that the athlete remains in the rounds in the tournament is an important criterion in revealing the role of the athlete in the network. The tour criterion used in this study as shown in (15) is obtained by summing the ratio of the number of rounds played by the athlete per year between the years determined and the total number of rounds in the tournament.

$$Tour = \frac{\sum_{\text{Year}=\text{The First Tournament Date}}^{\text{The Last Competition Year}} \frac{\text{The Number Of Rounds Played}_{\text{year}}}{\text{The Total Number Of Rounds}}}{\text{The Total Number Of Rounds}} \quad (15)$$

Here, the first competition year refers to the date of the first tournament on the network that will be weighted by the node, the last competition year refers to the date of the last tournament on the network that will be weighted by the node, the number of rounds played by the athlete refers to the number of rounds played by the athlete on a yearly basis, the total number of rounds refers to the total number of rounds held on a year. Although the number of criteria for calculating the weights of the nodes is completely independent, it is important to construct the parameters with distinctive features according to the structure and characteristics of the network.

After determining the criteria for the weights of the nodes, multi-criteria decision-making method APLOCO was carried out and the weights of the nodes were calculated based on the determined criteria. These weights could also be used to evaluate the importance and effectiveness of nodes in the network. After calculating the weights of the nodes, the nodes

in the center of the network have been revealed according to the proposed node weighting method.

#### IV. RESULTS AND FINDINGS

In the experimental study, a network of competitions between the years 2000-2017 of the Australian Open Tennis Tournament was established, and the network centrality criteria and the proposed node weighted centrality in this study were calculated. Calculated degree of centrality is shown in Table 1, Closeness centrality in Table 2, eigenvector centrality in Table 3, betweenness centrality in Table 4, and node-weighted centrality criteria are shown in Table 5.

Table 2: Degree Centrality

	Athlete	Degree
1	Federer R.	71
2	Ferrer D.	51
3	Djokovic N.	48
4	Murray A.	46
5	Berdych T.	45
6	Nadal R.	45
7	Roddick A.	43
8	Wawrinka S.	42
9	Hewitt L.	40
10	Tsonga J.W.	39

Table 2: Closeness Centrality

	Athlete	Closeness Centrality
1	Federer R.	0,460
2	Nadal R.	0,429
3	Ferrer D.	0,425
4	Djokovic N.	0,421
5	Roddick A.	0,420
6	Murray A.	0,418
7	Baghdatis M.	0,415
8	Berdych T.	0,413
9	Hewitt L.	0,410
10	Wawrinka S.	0,409

Table 3: Eigenvector Centrality

	Athlete	Eigenvector Centrality
1	Federer R.	1,000
2	Nadal R.	0,762
3	Djokovic N.	0,750
4	Ferrer D.	0,692

5	Murray A.	0,640
6	Wawrinka S.	0,636
7	Tsonga J.W.	0,602
8	Roddick A.	0,598
9	Baghdatis M.	0,598
10	Berdych T.	0,580

Table 4: Betweenness Centrality

	Athlete	Betweenness Centrality
1	Federer R.	0,1195
2	Ferrer D.	0,0547
3	Berdych T.	0,0462
4	Murray A.	0,0441
5	Hewitt L.	0,0424
6	Roddick A.	0,0414
7	Djokovic N.	0,0377
8	Nadal R.	0,0372
9	Lopez F.	0,0369
10	Tsonga J.W.	0,0342

Table 5: Node Weighting Centrality

	Athlete	Node Weighting Centrality
1	Federer R.	0,975
2	Djokovic N.	0,889
3	Nadal R.	0,869
4	Murray A.	0,857
5	Wawrinka S.	0,844
6	Berdych T.	0,839
7	Ferrer D.	0,837
8	Tsonga J.W.	0,837
9	Raonic M.	0,835
10	Agassi A.	0,833

Calculates network centrality according to the topological structure of the network; According to the results of Table 1, Table 2, Table 3 and Table 4, it is seen that the nodes, which have lost their importance over time, are in the centrality of the network. It has been observed that sportsmen such as Ferrer, Hewitt and Roddick, who have not played for a long time, are in the center of the network due to the high number of competitions in the past and still take place as popular nodes. However, it is important that these athletes, who have not played for a long time, have reduced their effectiveness and importance in the network and these situations can be identified when analyzing the network.

In the present study, it is seen that the weighted centrality results of the proposed node consist of the athletes who continue to play tennis as seen in Table 5. This indicates that effective and important nodes in the network can be detected. It is seen that the athletes who stopped playing tennis or who have lost their success in the early rounds, have moved away from the centrality of the network.

## V. CONCLUSION

In this study, node weighting was performed according to the criteria determined by multi-criteria decision-making method in a network composed of athletes in Australian Open tennis tournaments played between 2000-2017 and their competitions. The results of the network centrality calculated according to the obtained node weights were compared with the results of the network centrality criteria used in the complex networks. As a result of the experimental study, it was observed that the proposed node-weighted centrality criterion yielded more successful results compared to the network centrality measures that were calculated by topological measurements. Particularly in dynamic networks and time-varying conditions are included, more effective results have been obtained in determining the effective and important nodes in the network with the node-weighted centrality criterion. Especially in networks which are formed by different factors and which continue to grow, calculating network centrality with the topological structure of the network as well as the factors affecting the growth of the network, the network can be analyzed better.

## REFERENCES

- [1] S. H. Strogatz, *Exploring Complex Networks*, Nature, 410, (6825), 268-276 (2001).
- [2] Ş. Akal, "Gerçek ve model ağların karakteristik özelliklerinin karşılaştırılması", Marmara Üni. Öneri Dergisi., 11 (41): 251-272 (2014).
- [3] H.Kuduğ., "Sosyal Ağ Analizi Ölçütlerinin İş Ağlarına Uyarlanması", Yüksek Lisans Tezi, Ege Üniversitesi Fen Bilimleri Enstitüsü, İzmir, (2011).
- [4] C.Salman, "Yeni Bir Ağ Merkezilik Ölçütü: Göreceli Kenar Önemi Metodu", Yüksek Lisans Tezi, Hacettepe Üniversitesi Fen Bilimleri Enstitüsü, Ankara, (2018).
- [5] Lee, C. Y., "Correlations Among Centrality Measures In Complex Networks". arXiv preprint physics/0605220, (2006).
- [6] Wasserman, S., & Faust, K., "Social Network Analysis: Methods And Applications (Vol. 8)". Cambridge university press., (1994).
- [7] Bolland, J. M. . "Sorting Out Centrality: An Analysis Of The Performance Of Four Centrality Models In Real And Simulated Networks". Social networks, 10(3), 233-253, (1988).
- [8] Borgatti, S. P." Centrality And Network Flow". Social networks, 27(1), 55- 71, (2005).
- [9] McCulloh, I. "Detecting Changes In A Dynamic Social Network". Doctoral dissertation, Carnegie Mellon University, Pittsburgh (2009).
- [10] Rothenberg, R. B., Poterat, J. J., Woodhouse, D. E., Darrow, W. W., Muth, S. Q., & Klov Dahl, A. S. "Choosing A Centrality Measure: Epidemiologic Correlates In The Colorado Springs Study Of Social Networks". Social Networks, 17(3-4), 273-297, (1995).
- [11] Faust, K. (1997). "Centrality In Affiliation Networks". Social networks, 19(2), 157- 191, (1997).
- [12] Scott, J., "Social Network Analysis: A Handbook", London, Sage Publications, 209p., (2000).
- [13] Mika, P., "Social Networks And The Semantic Web, Semantic Web And Beyond Computing For Human Experience", Jain, R. and Sheth, A. (Eds.), New York, Springer Science and Business Media, 234p., (2007).
- [14] Newman, M.E.J., "Mathematics Of Networks", 12pp., The New Palgrave Dictionary of Economics, Durlauf, S.N. and Blume, L.E. (Eds.), Palgrave Macmillan, (2008).



- [15] Ruhnau, B., “*Eigenvector Centrality: A Node-Centrality?*”, Social Networks, 22 (4): 357-365,(2000).
- [16] T. Bulut, A New Multi Criteria Decision Making Method : Approach of Logarithmic Concept (APLOCO), Int. J. Artif. Intell. Appl. 9 (2018) 15–33. doi:10.5121/ijaia.2018.9102.
- [17] Tennis Results and Tennis Betting Data. ATP Men's Tour, Available: <http://www.tennis-data.co.uk/alldata.php>, (March, 2019)
- [18] Bastian M., Heymann S., Jacomy M. (2009). “*Gephi: an open source software for exploring and manipulating networks*”.International AAAI Conference on Weblogs and Social Media,(2009).